**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Answer:

We can infer that the time taken to service a car = 50 minutes.

Probability(x > 50) = 1 - Probability(x < 50).

Z = (x -*μ* )/*σ* = (x - 45)/8.0

Using the normal table,

Pr(x ≤ 50) = Pr(Z ≤ (50 - 45)/8.0) = Pr(Z ≤ 0.625)=73.4%

Probability that the service manager will not meet his demand will be = 100-73.4 = 26.6% or 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Answer: A)

False. This is because, if more employees at the processing center are older than 44, then that would result in pushing the mean towards 44. Since it is given that the mean age is 38, this is false.

B) True

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Answer:

As both X1 and X2 are independent normal random variables, X1 + X2 will be normal with parameters µ1 and µ2. Similarly, σ12 for X1 and σ22 for X2.

2X1 would result in scaling the normal distribution twice or 2 times.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Answer:

scipy.stats.norm.ppf(0.995,100,20) = 151.516586070978

(99.5th percentile)

scipy.stats.norm.ppf(0.005,100,20) = 48.483413929021985

(0.05th percentile)

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Answer:

(A)

scipy.stats.norm.ppf(0.025,45\*5,3) = 219.120

scipy.stats.norm.ppf(0.975,45\*5,3) = 230.879

scipy.stats.norm.ppf(0.025,45\*7,3) = 309.120

scipy.stats.norm.ppf(0.975,45\*7,3) = 320.879

So the Rupee range with 95% probability for the annual profit of the company is given by,

=P1 + P2

= [219.12, 230.87] + [309.12, 320.87]

= [528.24, 551.74]

Hence, we can say that profit of the company is in the range of [528.24, 551.74]

(B) Fifth percentile profit of the company can be said to be equal to 310.065 + 220.065 = 530.130

scipy.stats.norm.ppf(0.05,45\*7,3) = 310.065

scipy.stats.norm.ppf(0.05,45\*5,3) = 220.065

(C)

Division 2 having distribution N(7, 42)